To solve this equation, we find Green's function, $G(x;x',s)$ of (8.17):

$$\frac{\partial^2 G(x;x',s)}{\partial x^2} - \frac{s^2}{c^2} G(x;x',s) = 0 . \tag{8.19}$$

The general solution of (8.19) is given by:

$$G(x;x',s) = Ae^{-\frac{sx}{c}} + Be^{\frac{sx}{c}} .$$

The constants, A and B, are found from the boundary conditions and from the properties of Green's function. As $G(x;x',s)$ is finite for $x \to -\infty$ and $x \to +\infty$,

$$G_1 = Ae^{\frac{sx}{c}} \ \text{for} \ x < x'; \ G_2 = Be^{-\frac{sx}{c}} \ \text{for} \ x > x'.$$

$$\text{From (8.10a): } Ae^{\frac{sx}{c}} = Be^{-\frac{sx}{c}} \text{ ,i.e., } B = Ae^{\frac{2sx'}{c}} .$$

$$\text{From (8.10b): } A = -\frac{ce^{-\frac{sx'}{c}}}{2s}; \text{ hence, } B = -\frac{ce^{\frac{sx'}{c}}}{2s} .$$

$$G_1(x;x',s) = -\frac{c}{2s}\exp\left\{-\frac{s(x'-x)}{c}\right\} \quad \text{for } x<x' \tag{8.20a}$$

$$G_2(x;x',s) = -\frac{c}{2s}\exp\left\{\frac{s(x'-x)}{c}\right\} \quad \text{for } x>x' . \tag{8.20b}$$

By applying (8.10):

$$V(x,s) = \int_{-\infty}^{x} G_2(x;x',s)F(x',s)dx' + \int_{x}^{\infty} G_1(x;x',s)F(x',s)\,dx'$$

$$= V_1(x,s) + V_2(x,s). \tag{8.21}$$

Putting the value of $F(x,s)$ from (8.18), taking the inverse Laplace transform and considering the properties of the unit step function as well as the Dirac delta function, the solution for (8.9) is obtained for a rectangular return-stroke current. The results are given below:

$$V(x,t) = (V_{11} + V_{12} + V_{21} + V_{22})u(t - t_o) , \tag{8.22}$$