Duhamel Integral

$$R(t) = A(0)u(t) + \int_0^t \frac{\partial}{\partial \tau} A(\tau)u_r(t-\tau)d\tau. \tag{1.8}$$

This is Duhamel integral, where $u_r(t)$=unit step response, and R(t)=response due to the input A(t).

Duhamel integral has several forms in addition to that given in (1.8):

$$R(t) = A(0)u_r(t) + \int_0^t u_r(\tau)\frac{\partial}{\partial t} A(t-\tau)d\tau, \tag{1.8a}$$

$$R(t) = u_r(0)A(t) + \int_0^t \frac{\partial}{\partial \tau} u_r(\tau)A(t-\tau)d\tau\,, \tag{1.8b}$$

$$R(t) = u_r(0)A(t) + \int_0^t A(\tau)\frac{\partial}{\partial t} u_r(t-\tau)d\tau, \tag{1.8c}$$

$$R(t) = \frac{d}{dt}\int_0^t A(\tau)u_r(t-\tau)d\tau\,, \tag{1.8d}$$

$$R(t) = \frac{d}{dt}\int_0^t u_r(\tau)A(t-\tau)d\tau. \tag{1.8e}$$